Some Solved Problems

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Some problems of Laplace Transform.

1) Find the Laplace transform of

$$e^{-3t}(\cos 4t + 3\sin 4t)$$

Sol^n $L\{e^{-3t}(\cos 4t + 3\sin 4t)\}$

$$= L\{e^{-3t}\cos 4t\} + 3L\{e^{-3t}\sin 4t\}$$

$$= \frac{s+3}{(s+3)^2+4^2} + 3\cdot\frac{4}{(s+3)^2+4^2}$$

$$= \frac{s+15}{s^2+6s+25} \quad \underline{\text{Ans}}$$

$\left\{ \because L\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2+b^2} \right.$

$\left. L\{e^{at}\sin bt\} = \frac{b}{(s-a)^2+b^2} \right\}$

② Find $L\{t^3 e^{-3t}\}$

Sol^n Since $L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$

$$= \frac{n!}{s^{n+1}}$$

$$\therefore L(t^3) = \frac{3!}{s^4} = \frac{6}{s^4}$$

$$\therefore L(t^3 \cdot e^{-3t}) = \frac{6}{\{s-(-3)\}^4}$$

$$= \frac{6}{(s+3)^4}$$

$\left\{ \because L\{F(t)e^{at}\} = f(s-a) \right\}$

(3) Find out the Laplace transforms of $\sin^2 at$ and $\cos^2 at$.

Sol^n $\because \cos 2\theta = 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1$

$\therefore \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

& $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\therefore \sin^2 at = \dfrac{1-\cos 2at}{2}$

& $\sin^2 at = \dfrac{1+\cos 2at}{2}$

Now : $L\{\cos at\} = \dfrac{s}{s^2+a^2}$

$L\{\cos 2at\} = \dfrac{s}{s^2+4a^2}$

$\therefore L\{\sin^2 at\} = L\left\{\dfrac{1-\cos 2at}{2}\right\}$

$= \dfrac{1}{2}\{L(1) - L\cos(2at)\}$

$= \dfrac{1}{2}\left[\dfrac{1}{s} - \dfrac{s}{s^2+4a^2}\right]$

$= \dfrac{1}{2}\left[\dfrac{s^2+4a^2-s^2}{s(s^2+4a^2)}\right]$

$= \dfrac{2a^2}{s(s^2+4a^2)}$ Ans

(ii) $L\{\cos^2 at\} = L\left\{\dfrac{1}{2}(1+\cos 2at)\right\}$

$= \dfrac{1}{2}\left[L(1) + L(\cos 2at)\right]$

$= \dfrac{1}{2}\left[\dfrac{1}{s} + \dfrac{s}{s^2+4a^2}\right]$

$= \dfrac{s^2+2a^2}{s(s^2+4a^2)}$ Ans

(4) Evaluate $L\{F(t)\}$ if

$$F(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0 & 0 < t < 1 \end{cases}$$

Sol^n $L\{F(t)\} = \int_0^\infty F(t)\, e^{-st}\, dt$

$= \int_0^1 0 \cdot e^{-st}\, dt + \int_1^\infty (t-1)^2 e^{-st}\, dt$

$= 0 + \left[-\dfrac{(t-1)^2 e^{-st}}{s}\right]_{t=1}^{\infty} + \dfrac{2}{s}\int_1^\infty (t-1)\, e^{-st}\, dt$

$= 0 + 0 + \dfrac{2}{s}\left[-\dfrac{(t-1)\, e^{-st}}{s}\right]_{t=1}^{\infty}$

$= \dfrac{2}{s^2}\left(\dfrac{e^{-st}}{s}\right)_{t=1}^{\infty}$

$= \dfrac{2}{s^3} e^{-s}$ Ans